# A New Correlation of the Initial Collection Efficiency of Granular Aerosol Filtration

H. YOSHIDA and CHI TIEN

Department of Chemical Engineering and Materials Science
Syracuse University, Syracuse, NY 13210

Pendse and Tien (1982) proposed a correlation of the initial collection efficiency of granular filtration. Since the completion of that work, additional work in granular filtration has become available (Schmidt et al., 1978; Tardos et al., 1979; Thambimuthu, 1980; Mori and Iinoya, 1982; D'Ottavio and Goren, 1983; Walata, 1984). In light of these new data, modifications were introduced and an empirical correlation of the adhesive probability was developed to account for the effect due to particle bounce-off at high gas velocities.

The performance of granular filters is described by the unit collector efficiency, $\eta$. The particle concentration profile through a granular bed can be obtained from the solution of the following equation (Tien and Payatakes, 1979):

$$\frac{\partial c}{\partial z} = -\left[\frac{6(1-\epsilon)}{\pi}\right]^{1/3} \frac{\ln\left(\frac{1}{1-\eta}\right)}{dg} c \simeq -\left[\frac{6(1-\epsilon)}{\pi}\right]^{1/3} \frac{\eta}{dg} c \quad (1)$$

During the course of filtration, $\eta$ varies with time. The initial value of $\eta$ when the filter is relatively free of deposited particles is known as the initial or clean collector efficiency, $\eta_o$.

Over the years, several empirical correlations have been proposed (see Eqs. 24 and 25, Pendse and Tien, 1982). Moreover, three additional expressions were recently suggested (Thambinuthu, 1980; Goren, 1982; D'Ottavio and Goren, 1983). These different correlations were often established with the investigators' own data and did not contain the relevant parameters.

Pendse and Tien (1982) formulated a general consideration of $\eta_o$ from experimental as well as calculated results. $\eta_o$ is given as

$$(\eta_o)_{I,i} = (1 + 0.04 N_{Re_s}) \cdot \left[N_{St} + 0.48\left(4 - \frac{4N_R}{d_c^*}\right)^{1/2} \frac{N_R{}^{1.041}}{d_c^*}\right] \quad (2)$$

where the subscripts $I$ and $i$ denote interception and inertial impaction. Separate expressions due to gravitation $(\eta_o)_G$ and Brownian diffusion $(\eta_o)_D$ have also been developed (Rajagopalan and Tien, 1976). The addition of $(\eta_o)_D$ and $(\eta_o)_G$ to $(\eta_o)_{I,i}$ of Eq. 2 therefore provides an approximate estimate of $\eta_o$.

Equation 2 established the dependence of $(\eta_o)_{I,i}$ on $N_{Re_s}$ by observing the variation of the quantity $(\eta_o)_{I,i}/(\eta_o)_{I,i}$, $N_{Re_s} = 0$ vs. $N_{Re_s}$ based on trajectory calculation results. Equation 3 can be considered as

$$(\eta_o)_{I,i} = B(N_{Re_s})(\eta_o)_{I,i N_{Re_s}=0} \quad (3)$$

Before reexamining Eq. 2 it is important to note how $\eta_o$ can be obtained from experimental data. For a clean filter bed, $\eta$ is equal to $\eta_o$. By integrating Eq. 1, one has

$$\eta_o = \frac{dg}{L}\left(\frac{\pi}{6(1-\epsilon)}\right)^{1/3} \ln \frac{c_{in}}{c_{eff}} \quad (4)$$

The values of the influent and effluent concentrations can determine $\eta_o$. Similar expressions have also been used before. However, for consistency, the experimental values of $\eta_o$ used in this work are in accordance to Eq. 5.

## DEPENDENCE ON FLUID INERTIA

The data of D'Ottavio and Goren (1983) significantly extended the range of $N_{Re_s}$. Comparison between the data of D'Ottavio and Goren and Eq. 2 shows that Eq. 2 tends to overestimate. To obtain the dependence of $(\eta_o)_{I,i}$ on $N_{Re_s}$, Pendse and Tien first calculated the efficiency from interception $(\eta_o)_I$ using the constricted-tube model at different Reynolds numbers, $N_{Re_s}$ and $N_R$. By plotting $(\eta_o)_I/(\eta_o)_I$, $N_{Re3=0}$ vs. $N_{Re_s}$, the linear relationship $1 + 0.04\ N_{Re_s}$ was obtained. The same relationship was then assumed to be valid for $(\eta_o)_{I,i}/(\eta_o)_{I,i}\ N_{Re_s=0}$. The problem's reappraisal indicates that a better expression, valid for both low and high values of $N_{Re_s}$, is

$$B = \frac{(\eta_o)_{I,i}}{(\eta_o)_{I,i N_{Re_s}=0}} = 7 - 6\exp[-0.0065 N_{Re_s}] \quad (5)$$

## CORRECTION FOR THE EFFECT RESULTING FROM $N_R$ AND $N_{St}$

The new data mentioned earlier were obtained at much lower values of $N_R$ than those on which Eq. 2 was based. From Eqs. 2 and 3, one can see that the effect of $N_R$ is limited to the value of $\eta_o$ at $N_{Re_s} = 0$. Plotting $(\eta_o)_{I,i}/B$ vs. $N_{St}$ for the constant $N_R$ revealed that Eq. 2 agrees with data only as long as $N_R \geq 0.002$. A better representation of the results was found to be

$$\frac{(\eta_o)_{I,i}}{B} = 100\, N_{St}^2 + 0.19\left(4 - \frac{4N_R}{d_c^*} + \frac{N_R^2}{d_c^{*2}}\right)^{1/2} \frac{N_R^{1.041}}{d_c^*} \quad \times N_R \leq 0.002 \quad (6a)$$

$$= N_{St} + 0.48\left(4 - \frac{4N_R}{d_c^*} + \frac{N_{RR}^{\,2}}{d_c^{*2}}\right)^{1/2} \frac{N_R^{1.041}}{d_c^*} \quad \times N_R \geq 0.002 \quad (6b)$$

**TABLE 1. COMPARISONS OF DIFFERENT CORRELATIONS**

| Data Correlations | Doganoglu (1975) | Knettig and Beeckmans (1974) | Meltcher et al. (1978) | Mori and Iinoya (1981) | Schmidt et al. (1978) | Tardos et al. (1979) | D'Ottavio and Goren (1983) | Walata (1984) | Thambimuthu (1980) | All Data |
|---|---|---|---|---|---|---|---|---|---|---|
| D'Ottavio & Goren eq. | 46.9 | 33.3 | 33.3 | 20.0 | 40.3 | 32.0 | 70.9 | 33.3 | 59.6 | 53.1 |
| | 19.1 | 5.6 | 53.6 | 14.4 | 117.8 | 104.6 | 82.7 | 4.9 | 30.9 | 433.6 |
| Goren eq. | 40.6 | 66.7 | 16.7 | 20.0 | 40.3 | 42.0 | 41.8 | 83.3 | 68.1 | 44.4 |
| | 38.7 | 6.0 | 77.8 | 11.8 | 85.9 | 109.2 | 172.7 | 1.0 | 22.3 | 525.4 |
| Present work | 71.9 | 33.3 | 38.9 | 60.0 | 43.5 | 78.0 | 50.4 | 83.3 | 68.1 | 56.9 |
| | 22.6 | 5.9 | 18.4 | 6.2 | 54.6 | 26.6 | 110.7 | 2.9 | 24.2 | 272.1 |
| Pendse & Tien eq. | 71.9 | 0.0 | 38.9 | 80.0 | 51.6 | 73.9 | 38.3 | 83.3 | 70.2 | 53.1 |
| | 23.0 | 8.4 | 13.0 | 1.3 | 57.4 | 29.5 | 242.4 | 2.9 | 23.0 | 400.9 |
| Paretsky eq. | 65.6 | 100.0 | 44.4 | 40.0 | 37.1 | 48.0 | 39.7 | 50.0 | 72.3 | 48.2 |
| | 11.9 | 0.2 | 53.0 | 5.0 | 81.2 | 32.7 | 324.9 | 5.6 | 20.3 | 534.8 |
| Meisen Eq. | 78.1 | 33.3 | 27.8 | 100.0 | 53.2 | 72.0 | 37.6 | 83.3 | 76.6 | 54.5 |
| | 18.7 | 5.2 | 28.6 | 0.9 | 32.6 | 28.9 | 198.1 | 2.3 | 17.2 | 332.5 |
| Meltcher eq. | 68.8 | 100.0 | 33.3 | 20.0 | 46.8 | 50.0 | 49.6 | 50.0 | 70.2 | 53.1 |
| | 12.8 | 0.2 | 35.2 | 7.2 | 57.6 | 31.7 | 179.8 | 6.5 | 20.4 | 351.4 |
| Doganoglu eq. | 100.0 | 0.0 | 61.1 | 80.0 | 67.7 | 56.0 | 41.1 | 16.7 | 40.4 | 53.1 |
| | 4.0 | 22.2 | 21.0 | 1.5 | 30.1 | 41.8 | 211.9 | 14.8 | 75.6 | 422.9 |

## CONTRIBUTION RESULTING FROM BROWNIAN DIFFUSION

Equation 2 was derived on the basis that inertial impaction and interception are the dominant collection mechanisms. To account for the Brownian diffusion contribution, a combination of Eq. 2 and $(\eta_o)_D$ can be used to estimate $\eta_o$.

The recently available experimental data allow a validity test of this combination over a wider range of the Peclet number. The results indicate that a linear combination of Eq. 2 and the expression of Rajagopalan and Tien (1976) does not agree well with experiments, especially for high values of $N_{Pe}$. Significant improvement was observed when the $(\eta_o)_D$ expression of Tardos et al. (1979) was used, or

$$(\eta_o)_D = 4\left(\frac{1.13}{\epsilon^{1/2}}\right) N_{Pe}^{-1/2},\ N_{Re_s} > 30 \qquad (7)$$

In sum, $\eta_o$ may be taken to be the value of $(\eta_o)_{I,i}$ (from Eqs. 6a and 6b), $(\eta_o)_D$ (from Eq. 7), or estimated in accordance with the suggestion of Rajagopalan and Tien (1976).

Calculations were made to test the validity of the new correlation by comparing it against experimental data and earlier correlations. In Table 1, two figures are given that correspond to correlation expressions and experimental results. The first figure gives the percentage of data points reported in a study that can be predicted within a factor of 2 for each of the correlations listed. The second figure refers to values of $[\Sigma(\ln(\eta_o)_{cal} - \ln(\eta_o)_{exp}]^2$, which were calculated for each correlation expresssion. The improvement introduced by the new correlation is obvious.

The unit collector efficiency increases with the increase of the particle inertia characterized by $N_{St}$. As $N_{St}$ increases beyond a certain value, $\eta_o$, however, decreases. Both the data of D'Ottavio and Goren (1983) and of Tardos et al. (1979) displayed this be-

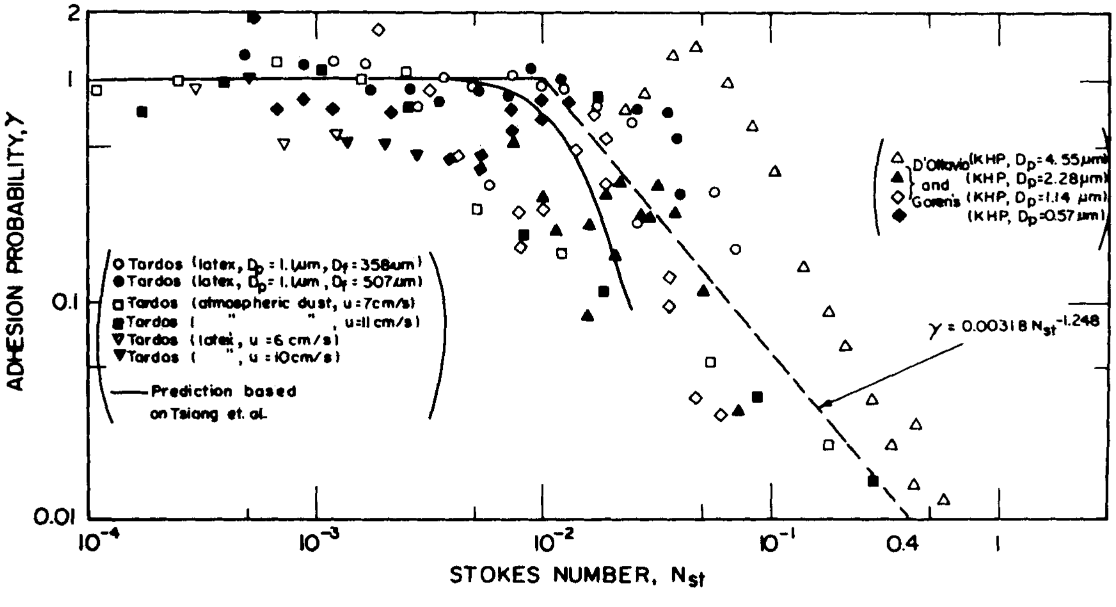


Figure 1. Adhesion probability as a function of $N_{St}$.

havior, which can be attributed to the bounce-off of impacting aerosol particle from filter grains. An adhesion probability, $\gamma$, can be defined as

$$\gamma = (\eta_o)_{exp}/(\eta_o)_{transp} \quad (8)$$

where $(\eta_o)_{transp}$ is the collection efficiency if all the impacting particles are collected. Since the developed correlation does not include the effect of possible particle bounce-off, the value of $\eta_o$ predicted from Eqs. 6a and 6b is $(\eta_o)_{transp}$. A possible assessment of the particle inertia vs. particle adhesion effect is made by plotting $(\eta_o)_{exp}/(\eta_o)_{transp}$ vs. $N_{St}$ into the region when the decrease in $\eta_o$ with the increase of gas velocity is observed.

This situation was plotted and is shown in Figure 1. The data of Tardos et al. (1979) and D'Ottavio and Goren (1983) are included. At low $N_{St}$ the values of $(\eta_o)_{exp}/(\eta_o)_{transp}$ were found to scatter around 1. The decrease in $\eta_o$ occurs at $N_{St} = 0.01$. For $N_{St} \geq 0.01$, the adhesion probability can be represented as

$$\gamma = 0.00318 N_{St}^{-1.248} \quad (9)$$

Tsiang et al. (1982) incorporated the concept of the escape velocity for particle bounce-off in a study of aerosol collection in model filters. Their results can be extended to granular filtration if the material constants of their experimental systems (polyvinyl toulene latex particles and Evanohm alloy fiber) are similar to those used by D'Ottavio and Goren and by Tardos et al. The estimated $\gamma$ vs. $N_{St}$ curve with this approach is also shown in Figure 1 and is in reasonable agreement with experimental data. The critical Stokes number was $4 \times 10^{-3}$ and the estimated values of $\gamma$ were lower than those given in Eq. 15. That the two entirely different approaches yield similar conclusions adds further credence to the result of this study.

## NOTATION

- $B$ = parameter defined by Eq. 6
- $c$ = particle concentration
- $C_s$ = Cunningham's slip factor
- $c_{in}$, $c_{eff}$ = influent and effluent particle concentrations
- $d_c^*$ = dimensionless constriction diameter of constricted tube, $d_c/d_g$, where $d_c$ is the average pore constriction diameter
- $d_g$ = grain diameter
- $d_p$ = particle diameter
- = length of unit bed element
- $L$ = filter height
- $N_{Pe}$ = Peclet number $d_g V_s/D_{BM}$, $D_{BM}$ Brownian diffusivity
- $N_R$ = relative parameter or interception parameter, $d_p/d_g$
- $N_{Re,s}$ = fluid Reynolds number, $d_g V_s \rho/\mu$
- $N_{St}$ = Stokes number, $d_p^2 V_s \rho_p C_s/9\mu d_g$
- $V_s$ = superficial fluid velocity
- $z$ = axial distance, bed depth

### Greek Letters

- $\epsilon$ = bed porosity
- $\lambda$ = filter coefficient
- $\eta$ = unit collector efficiency
- $\mu_f$ = fluid viscosity
- $\rho$ = fluid density
- $\rho_p$ = particle density
- $\gamma$ = adhesion probability

### Subscripts

- $o$ = initial value
- $D,G,I,i$ = Brownian diffusion, gravitation, interception, and inertia, respectively

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